Double-exciton component of the cyclotron spin-flip mode in a quantum Hall ferromagnet

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We report on the study of the cyclotron spin-flip excitation (CSFE) in a quantum Hall system at unit filling, where both the cyclotron quantum number and the spin number are changed by one compared to the ground state. The CSFE composite mode has a double-exciton component, which, even within the first-order approximation in terms of the interaction energy, contributes to the CSFE correlation shift measured from the combined cyclotron and Zeeman gap. This component cannot be accounted for in the single-mode approach, and the problem effectively becomes the quantum four-body one. The final result is obtained for zero total momentum. Since in that case, the excitation is optically active, the result is compared with available experimental data.

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I. INTRODUCTION

A two-dimensional electron gas (2DEG) in a high perpendicular magnetic field possesses many remarkable features.[1] In particular, it presents a rare case of strongly correlated system governed by *real* Coulomb interaction (not by a model Hamiltonian), where, nevertheless, some solutions of the quantum many-body problem can be found exactly. Indeed, under the conditions of integer quantum Hall effect (when the filling factor is $\nu=1,2,3,\ \ldots$), the one-cyclotron magnetoplasma and the lowest spin-flip modes are calculated analytically to the leading order in the parameter $r_c=E_C/\hbar\omega_c$.[2–5] ω_c is the cyclotron frequency; $E_C=\alpha e^2/\kappa l_B$ is the characteristic interaction energy, with α being the average form factor related to the finite thickness of the 2DEG ($0.3\leqslant\alpha<1$); and l_B is the magnetic length. This astounding property is the feature of either filled or half-filled highest-occupied Landau level (LL), where the simplest-type excitations are single excitons or superposition of single-exciton modes. The many-body problem is thereby reduced to the two-body one, i.e., to the interaction of electron with an effective hole. Being quite in the context of similar studies, the present paper, however, concerns the case which cannot be reduced to a single-exciton problem.

We recall that 2DEG excitons are characterized by sublevels $a=(n_a,\sigma_a)$ and $b=(n_b,\sigma_b)$, where electron is promoted from the n_ath LL with spin component $S_z=\sigma_a$ to the n_bth LL with $S_z=\sigma_b$. The relevant quantum numbers are $\delta n=n_b-n_a$, $\delta S_z=\sigma_b-\sigma_a$, and the two-dimensional (2D) wave vector $\mathbf{q}$. An excited state is reduced to a single exciton and becomes exactly solvable in the following cases: (i) at odd filling ν, when $\delta n=1$ and $\delta S_z=0$ (magnetoplasmon) or $\delta n=0$ and $\delta S_z=-1$ (spin wave) (Refs. 2, 4, and 6) and (ii) at even ν, when $\delta n=1$ and $\delta S_z=0,\pm1$ (magnetoplasmon and spin-flip triplet).[4–6] At the same, other excitations may be discussed within an approximate single-mode (SM) approach [in some publications, called "time-dependent Hartree-Fock (HF)" approximation[7,8]], which excludes any quantum fluctuations from a single exciton to a double- or many-exciton states. For the above simplest cases of δn and δS_z, the SM calculation gives an asymptotically exact result, which may be found perturbatively to the first order in r_c (Ref. 9) because these $(\delta n,\delta S_z)$ sets cannot correspond to any states except single-exciton modes. Any complication of $(\delta n,\delta S_z)$ makes the calculations substantially more difficult due to the necessary expansion of the basis to the entire continuous set of many-exciton states with the same total numbers δn, δS_z, and $\mathbf{q}$. For example, the double-cyclotron plasmon with $\delta n=2$, $\delta S_z=0$, and with given $\mathbf{q}$, "dissociates" into double-exciton states consisting of one-cyclotron plasmon's pairs with the total momentum equal to $\mathbf{q}$.[4] At odd ν, a similar dissociation occurs for the cyclotron spin-flip excitation (CSFE), where $\delta n=-\delta S_z=1$. The proper double-exciton states are pairs of a magnetoplasmon $(\delta n=1,\ \delta S_z=0)$ and a spin wave $(\delta n=0,\ \delta S_z=-1)$. The problem thus changes from the two-body case to the *four-body* one, and the correct solution should be presented in the form of the combination of the single-exciton mode and continuous set of double-exciton states.[6] It is important that in both cases, the desired solution corresponds to a discrete line against the background of a continuous spectrum of free exciton pairs. The technique of correct solution has to be of essentially non-Hartree-Fock (non-HF) type. Actually, this work concerns the fundamental question of consistency of the HF approach.

By considering the case of unit filling factor, where the number of electrons is equal to the number of magnetic-flux quanta N_ϕ, we now report on a study of the CSFE with $\mathbf{q}=0$. This state is optically active and identified in the inelastic light scattering experiments.[10,11] Besides, it is exactly this spin-flip magnetoplasma mode, which is the key component of the elementary perturbation used in the microscopic approach to the skyrmionic problem.[12] The calculation is performed in a "quasianalytical" way, which should, in principle, lead to the result that is exact in the leading approximation in r_c. In our case, the envelope function determining the combination of the double-exciton states is one dimensional—i.e., it only depends on the modulus of the excitons' relative momentum. This function is chosen in the form of expansion over infinite orthogonal basis, where every basis vector obeys a specific symmetry condition necessary for the total envelope function. Even to the *first-order approximation* in r_c, we obtain a double-exciton correction to the former SM result[8,10] for the CSFE energy.

II. EXCITONIC REPRESENTATION FORMALISM

As a technique, we use the *excitonic representation* (ER), which is a convenient tool for the description of the 2DEG in a perpendicular magnetic field.[5,6,13] When acting on the vacuum $|0\rangle$ (in our case

$$|0\rangle = |\overbrace{\uparrow,\uparrow,\ \ldots\uparrow}^{N_\phi}\rangle),$$

the exciton operators produce a set of basis states, which diagonalize the single-particle term of the Hamiltonian and some part $\hat{H}_{\rm ED}$ of the interaction Hamiltonian.[6,12] Exciton states are classified by $\mathbf{q}$ and it is essential that in this basis the LL degeneracy is lifted.

So, the generic Hamiltonian is $\hat{H}=\hat{H}_1+\hat{H}_{\rm int}$, where

$$\hat{H}_1 = \sum_\sigma \int d\mathbf{r}\hat{\Psi}^\dagger_\sigma(\mathbf{r})\left[\frac{1}{2m^*}(i\vec{\nabla}-e\vec{A}/c)^2 + g\mu_B B\hat{S}_z\right]\hat{\Psi}_\sigma(\mathbf{r})$$

and

$$\hat{H}_{\rm int} = \frac{1}{2}\sum_{\sigma_1,\sigma_2}\int d\mathbf{r}_1 d\mathbf{r}_2 \hat{\Psi}^\dagger_{\sigma_2}(\mathbf{r}_2)\hat{\Psi}^\dagger_{\sigma_1}(\mathbf{r}_1) \times U(\mathbf{r}_1-\mathbf{r}_2)\hat{\Psi}_{\sigma_1}(\mathbf{r}_1)\hat{\Psi}_{\sigma_2}(\mathbf{r}_2). \quad (1)$$

Choosing, for example, the Landau gauge and substituting for the Schrödinger operator $\hat{\Psi}^\dagger_\sigma=\Sigma_{np}a^\dagger_{np\sigma}\psi^*_{np\sigma}$ (indexes n,p,σ label the LL number, intra-LL state, and spin sublevel), one can express the Hamiltonian (1) in terms of combinations of the various components of the density-matrix operators.[5,6,12] These are exciton operators defined as[5,6,12,13]

$$\mathcal{Q}^\dagger_{ab\mathbf{q}} = N_\phi^{-1/2}\sum_p e^{-iq_x p}b^\dagger_{p+q_y/2}a_{p-q_y/2} \quad \text{and} \quad \mathcal{Q}_{ab\mathbf{q}} = \mathcal{Q}^\dagger_{ba-\mathbf{q}}, \quad (2)$$

and obeying the commutation algebra[6]

$$[\mathcal{Q}^\dagger_{cd\mathbf{q}_1},\mathcal{Q}^\dagger_{ab\mathbf{q}_2}] \equiv N_\phi^{-1/2}[e^{-i(\mathbf{q}_1\times\mathbf{q}_2)_z/2}\delta_{b,c}\mathcal{Q}^\dagger_{ad\mathbf{q}_1+\mathbf{q}_2} - e^{i(\mathbf{q}_1\times\mathbf{q}_2)_z/2}\delta_{a,d}\mathcal{Q}^\dagger_{cb\mathbf{q}_1+\mathbf{q}_2}], \quad (3)$$

(in our units $l_B=\sqrt{c\hbar/eB}=1$). Here $a,b,c,\ \ldots$ are binary indexes (see above), which means that $a^\dagger_p=a^\dagger_{n_a p\sigma_a}$ and $b^\dagger_p=a^\dagger_{n_b p\sigma_b}\ldots$. We will also employ for binary indexes the notations $n=(n,\uparrow)$ and $\bar{n}=(n,\downarrow)$, so that the single-mode component of the CSFE is defined as $\mathcal{Q}^\dagger_{01\bar{\mathbf{q}}}|0\rangle$. The interaction Hamiltonian can be presented as $\hat{H}_{\rm int}=\hat{H}_{\rm ED}+\hat{H}'$, where $\hat{H}_{\rm ED}$, if applied to the state $\mathcal{Q}^\dagger_{ab\mathbf{q}}|0\rangle$ yields a combination of single-exciton states with the same numbers δn, δS_z, and $\mathbf{q}$ (see Refs. 6 and 12 and therein $\hat{H}_{\rm ED}$ expressed in terms of exciton operators). In the framework of the above SM approximation, the CSFE correlation energy[10] is obtained from the equation $\mathcal{E}_{01\bar{}}(q)=\langle 0|\mathcal{Q}_{01\bar{}\mathbf{q}}[\hat{H}_{\rm int},\mathcal{Q}^\dagger_{01\bar{}\mathbf{q}}]|0\rangle$, where only the $\hat{H}_{\rm ED}$ part of the interaction Hamiltonian contributes to the expectation. In the following, we need this SM value at $q=0$, namely $\mathcal{E}_{01\bar{}}(0)\equiv\mathcal{E}_{\rm SM}=\frac{1}{2}\int_0^\infty p^3dpV(p)e^{-p^2/2}$, where $2\pi V(q)$ is the Fourier component of the effective Coulomb vertex in the layer. In the strictly 2D limit, $\alpha\to 1$ and $V(q)\to e^2/\kappa l_B q$.

The problem arises due to the "troublesome" part $\hat{H}'$ of the interaction Hamiltonian, which cannot be diagonalized in terms of single-exciton states. For our task, we keep in $\hat{H}'$ only the terms contributing to $[\hat{H}',\mathcal{Q}^\dagger_{01\bar{}\mathbf{q}}]|0\rangle$ and besides preserving the cyclotron part of the total energy (i.e., commuting with $\hat{H}_1$). In terms of the ER, these are[5,6]

$$\hat{H}_{01\bar{}} = \sum_{\mathbf{q}}\frac{q^2}{2}V(q)e^{-q^2/2}\mathcal{Q}^\dagger_{01\mathbf{q}}\mathcal{Q}_{01\bar{}\mathbf{q}} + \text{H.c.} \quad (4)$$

Using Eq. (3) and identities $\mathcal{Q}^\dagger_{aa\mathbf{q}}|0\rangle\equiv N_\phi^{-1/2}\delta_{\mathbf{q},0}|0\rangle$ if $a=(0,\uparrow)$ and $\mathcal{Q}^\dagger_{aa\mathbf{q}}|0\rangle\equiv 0$ if $a\neq(0,\uparrow)$, one can find that the operation of $\hat{H}_{01\bar{}}$ on vector $\mathcal{Q}^\dagger_{01\bar{}\mathbf{q}}|0\rangle$ results in a combination of states of the type of $N_\phi^{-1/2}\Sigma_{\mathbf{s}}f(\mathbf{s})\mathcal{Q}^\dagger_{00\bar{}\mathbf{q}/2-\mathbf{s}}\mathcal{Q}^\dagger_{01\mathbf{q}/2+\mathbf{s}}|0\rangle$, with a certain regular and square-integrable envelope function $\int|f(\mathbf{s})|^2d\mathbf{s}\sim 1$. The norm of this combination is not small as compared to $\langle 0|\mathcal{Q}_{01\bar{}\mathbf{q}}\mathcal{Q}^\dagger_{01\bar{}\mathbf{q}}|0\rangle\equiv 1$ and the term (4) must be taken into account when calculating the CSFE energy.

On the other hand, if the set of double-exciton states $|\mathbf{s},\mathbf{q}\rangle=\mathcal{Q}^\dagger_{00\bar{}\mathbf{q}/2-\mathbf{s}}\mathcal{Q}^\dagger_{01\mathbf{q}/2+\mathbf{s}}|0\rangle$ is considered, then one finds that they, first, are not exactly but "almost" orthogonal, $\langle\mathbf{q}_1,\mathbf{s}_1|\mathbf{s}_2,\mathbf{q}_2\rangle=\delta_{\mathbf{q}_1,\mathbf{q}_2}\{\delta_{\mathbf{s}_1,\mathbf{s}_2}\}$, where $\{\delta_{\mathbf{s}_1,\mathbf{s}_2}\}\equiv\delta_{\mathbf{s}_1,\mathbf{s}_2}-e^{i(\mathbf{s}_1\times\mathbf{s}_2)_z}/N_\phi$ and, second, $|\mathbf{s},\mathbf{q}\rangle$ satisfies the equation

$$[\hat{H}_{\rm int},\mathcal{Q}^\dagger_{00\bar{}\mathbf{q}/2-\mathbf{s}}\mathcal{Q}^\dagger_{01\mathbf{q}/2+\mathbf{s}}]|0\rangle = [\mathcal{E}_{\rm sw}(|\mathbf{q}/2+\mathbf{s}|)+\mathcal{E}_{\rm mp}(|\mathbf{q}/2-\mathbf{s}|)]|\mathbf{s},\mathbf{q}\rangle + |\tilde{\varepsilon}\rangle, \quad (5)$$

where the state $|\tilde{\varepsilon}\rangle$ has a negligibly small norm $\langle\tilde{\varepsilon}|\tilde{\varepsilon}\rangle\sim E_C/N_\phi$. Therefore, the double-exciton state $|\mathbf{s},\mathbf{q}\rangle$ in the thermodynamic limit actually corresponds to the free noninteracting excitons, one of them is a spin exciton (spin wave) with energy $|g\mu_B B|+\mathcal{E}_{\rm sw}$, where

$$\mathcal{E}_{\rm sw}(q) = \int_0^\infty pdpV(p)e^{-p^2/2}[1-J_0(pq)], \quad (6)$$

while the other is a magnetoplasmon with energy $\hbar\omega_c+\mathcal{E}_{\rm mp}$, where

$$\mathcal{E}_{\rm mp}(q) = \frac{q^2}{2}V(q)e^{-q^2/2} + \int_0^\infty pdpe^{-p^2/2}V(p)\left(1-\frac{p^2}{2}\right) \times[1-J_0(pq)], \quad (7)$$

where J_0 is the Bessel function (cf. Refs. 2 and 4).

Thus, for the CSFE state, we try the vector $|X_{\mathbf{q}}\rangle=\hat{X}_{\mathbf{q}}|0\rangle$, where $\hat{X}_{\mathbf{q}}$ is a combined operator,

$$\hat{X}_{\mathbf{q}} = \mathcal{Q}^\dagger_{01\bar{}\mathbf{q}} + \frac{1}{\sqrt{2N_\phi}}\sum_{\mathbf{s}}\varphi_q(\mathbf{s})\mathcal{Q}^\dagger_{00\bar{}\mathbf{q}/2-\mathbf{s}}\mathcal{Q}^\dagger_{01\mathbf{q}/2+\mathbf{s}}. \quad (8)$$

Actually, only a certain "antisymmetrized" part $\{\varphi_q\}$ of the envelope functions contributes to the double-exciton combination in $|X_{\mathbf{q}}\rangle$.[3,5,6] In our case, the antisymmetry transform is $\{\varphi_q\}=\varphi_q(\mathbf{s})-\frac{1}{N_\phi}\Sigma_{\mathbf{s}'}e^{i(\mathbf{s}\times\mathbf{s}')_z}\varphi_q(\mathbf{s}')$. Such a specific feature originates from the generic permutation antisymmetry of the Fermi wave function of our many-electron system. We may

therefore consider only the "antisymmetric" functions, for which

$$\varphi_q = \{\varphi_q\}/2. \tag{9}$$

Our task is to find the energy of the eigenvector $|X_{\mathbf{q}}\rangle$ and the "wave function" $\varphi_q(\mathbf{s})$, assuming that the latter is regular and square integrable. If E_q is the correlation part of the total CSFE energy (namely, $E_{\mathrm{CSFE}}=E_{\mathrm{vac}}+|g\mu_B B|+\hbar\omega_c+E_q$), then E_q is found from

$$[\hat{H}_{\mathrm{ED}}+\hat{H}'_{0\bar{1}},\hat{X}_{\mathbf{q}}]|0\rangle = E_q|X_{\mathbf{q}}\rangle \tag{10}$$

(see Appendix A). Now we project this equation onto two basis states $|\mathbf{p},\mathbf{q}\rangle$ and $\mathcal{Q}^{\dagger}_{\bar{0}1\mathbf{q}}|0\rangle$, and obtain two closed coupled equations,

$$(2N_\phi)^{1/2}\langle \mathbf{q},\mathbf{p}|[\hat{H}'_{01},\mathcal{Q}^{\dagger}_{\bar{0}1\bar{0}\mathbf{q}}]|0\rangle + \sum_{\mathbf{s}} \varphi_q(\mathbf{s})\langle \mathbf{q},\mathbf{p}|[\hat{H}_{ED},\mathcal{Q}^{\dagger}_{\bar{0}\bar{0}\mathbf{q}/2-\mathbf{s}}\mathcal{Q}^{\dagger}_{01\mathbf{q}/2+\mathbf{s}}]|0\rangle = E_q\varphi_q(\mathbf{p}), \tag{11}$$

and

$$\mathcal{E}_{0\bar{1}}(q) + (2N_\phi)^{-1/2}\sum_{\mathbf{s}} \varphi_q(\mathbf{s}) \times\langle 0|\mathcal{Q}_{0\bar{1}\mathbf{q}}[\hat{H}'_{01},\mathcal{Q}^{\dagger}_{\bar{0}\bar{0}\mathbf{q}/2-\mathbf{s}}\mathcal{Q}^{\dagger}_{01\mathbf{q}/2+\mathbf{s}}]|0\rangle = E_q, \tag{12}$$

for E_q and $\varphi_q(\mathbf{p})$.

III. EIGENSTATES, THE CYCLOTRON SPIN-FLIP EXCITATION ENERGY, AND CONTINUOUS SPECTRUM

Next step is a routine treatment of Eqs. (11) and (12) in terms of calculation of commutators guided by commutation rules (3). In the $\mathbf{q}=0$ case, which we immediately consider, the function $\varphi_0(\mathbf{p})$ depends only on the modulus of $\mathbf{p}$. As a result, we obtain (cf. Appendix B)

$$[E-\mathcal{E}_{\mathrm{sw}}(q)-\mathcal{E}_{\mathrm{mp}}(q)]\varphi(q)+\int_0^\infty sds\left[K_1(s,q)\varphi(s) + \frac{K_2(s)}{\pi}\int_0^\pi d\phi(1-\cos[\mathbf{s}\times\mathbf{q}])\varphi(|\mathbf{q}+\mathbf{s}|)\right] = g(q), \tag{13}$$

and

$$E-\mathcal{E}_{\mathrm{SM}} = \frac{1}{\sqrt{2}}\int_0^\infty dpp^3V(p)e^{-p^2/2}\varphi(p), \tag{14}$$

(we omit subscript 0 in E_0 and φ_0), where

$$g(q) = \frac{q^2}{2\sqrt{2}}V(q)e^{-q^2/2} - \frac{1}{2\sqrt{2}}\int_0^\infty p^3V(p)e^{-p^2/2}J_0(pq)dp, \tag{15}$$

$$K_1(q,s) = \frac{s^2}{2}e^{-s^2/2}V(s)J_0(qs),$$

and

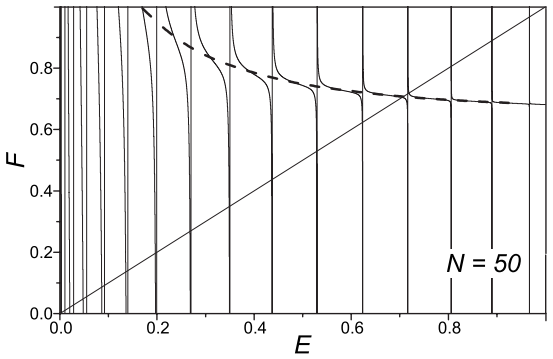

FIG. 1. Graphical solution of Eqs. (13) and (14). Intersection of the $F=E$ straight line with the dashed line corresponds to the CSFE energy $E_{\mathrm{SF}}\approx 0.71$. See text for details.

$$K_2(s) = \left(2-\frac{s^2}{2}\right)V(s)e^{-s^2/2} \tag{16}$$

[ϕ in Eq. (13) is the angle between $\mathbf{s}$ and $\mathbf{q}$].

The problem has, thus, been integrable to yield in the thermodynamic limit a pair of coupled integral equations for one-dimensional function $\varphi(q)$ and the eigenvalue E. In order to solve this system, we employ the method of expansion in orthogonal functions,

$$\varphi(q) = \sum_{n=1,3,5,\ \ldots}^{2N-1} A_n\psi_n(q). \tag{17}$$

These $\psi_n=\sqrt{2}L_n(q^2)e^{-q^2/2}$, with odd indexes of the Laguerre polynomials ($\int_0^\infty qdq\psi_m\psi_n=\delta_{m,n}$), are chosen as a natural basis satisfying, (i) the property of integrability and expected analytic and asymptotic features of $\varphi_0(q)$ and (ii) the antisymmetry condition (9). In other words, we change from the basis formed by the set of nonorthogonal double-exciton states $|\mathbf{s},0\rangle\equiv\mathcal{Q}^{\dagger}_{\bar{0}\bar{0}-\mathbf{s}}\mathcal{Q}^{\dagger}_{01\mathbf{s}}|0\rangle$ to a new set of basis states $|\mathrm{DX},n\rangle=(2N_\phi)^{-1/2}\Sigma_{\mathbf{s}}\psi_n(s)|\mathbf{s},0\rangle$, which are strictly orthogonal. Indeed, one can check by employing Eq. (3) and identity $\frac{1}{N_\phi}\Sigma_{\mathbf{s}}e^{i(\mathbf{q}\times\mathbf{s})_z}\psi_n(s)\equiv\int_0^\infty sdsJ_0(qs)\psi_n(s)\equiv-\psi_n(q)$ that $\langle m,\mathrm{DX}|\mathrm{DX},n\rangle\equiv\delta_{m,n}$. The integer number N is dimensionality of this new double-exciton basis.

After substitution of Eq. (17) into Eq. (14), the latter takes the form $E=F$, where

$$F = \mathcal{E}_{\mathrm{SM}} + \frac{1}{\sqrt{2}}\sum_{n=1,3,5,\ \ldots}^{2N-1} A_n\int_0^\infty dpp^3V(p)e^{-p^2/2}\psi_n(p). \tag{18}$$

Let us consider the ideal 2D case, where $V(q)=1/q$. Here and below, energy is measured in units of $e^2/\kappa l_B$. After substitution of the expansion (18) into Eq. (13), further multiplication by basis functions $\psi_m(q)$ and integration ($\int\ldots qdq$) lead to the set of N linear algebraic equations with respect to A_n. Finding A_n for a given E and substituting them into Eq. (18), we obtain $F(E)$. The condition $F(E)=E$ yields the desired result $E=E_{\mathrm{SF}}$.

Figure 1 shows the result of calculations for $N=50$. The lines, which are restricted by vertical asymptotes, reflect the result of calculation of $F(E)$. Points of singularity $E^{(i)}$, at which F goes to infinity are roots of the equation $D_N(E)=0$,

where D_N is the determinant corresponding to the "left side" of the set of equations for A_n. By increasing N, we increase the order of equation $D_N(E)=0$, so that this has up to N real roots. Indeed, when observing the evolution of $F(E)$ with increasing N, one finds that the number of singular points grows, and they become more densely placed. For $N\to\infty$, one could expect that a singular point appears within an arbitrarily small vicinity of every value E. Since all the vertical asymptotes $E=E^{(i)}$ are crossed by the straight line $F=E$ (see Fig. 1), we come to the conclusion that for any E, there is a singular solution of Eqs. (13) and (14). Such solutions with singular functions $\varphi(q)$ form a band.

The physical meaning of this result is quite transparent. Namely, the band corresponds to energy $\mathcal{E}_{\rm sw}(q)+\mathcal{E}_{\rm mp}(q)$ of unbound spin wave and magnetoplasmon, which almost do not scatter at each other [see Eq. (5)]. Indeed, considering that $q_0(E)$ is the root of the equation $\mathcal{E}_{\rm sw}(q_0)+\mathcal{E}_{\rm mp}(q_0)=E$ and substituting $\varphi_0(\mathbf{s})=\sqrt{N_\phi}\delta_{|\mathbf{s}|,q_0}$ into Eq. (8), we find the combined state $|X_0\rangle$, which presents itself the two-exciton component with the norm $\sim q_0\sqrt{N_\phi}$, and a minute admixture of the component $\mathcal{Q}^\dagger_{0\bar{1}0}|0\rangle$ having unit norm. It would be interesting to see how this result follows from Eqs. (13) and (14). For that, we change in the thermodynamic limit $\sqrt{N_\phi}\delta_{|\mathbf{s}|,q_0}\to\sqrt{8/\pi}\delta(s-q_0)$ and isolate the singularity $\varphi(q)=C\delta(q-q_0)+u(q)$, where $u(q)$ is assumed to be regular. By substituting this expression for $\varphi(q)$ into Eqs. (13) and (14), we obtain, at any given parameter E within the band $0<E<\mathcal{E}_{\rm sw}(\infty)+\mathcal{E}_{\rm mp}(\infty)$, two coupled equations determining constant $C\sim 1$ and a regular function $u(q)$.

Now, turning back to Fig. 1, we consider the solution $E=F(E)$, where the $F=E$ line crosses a conventional envelope curve tracing the regions of regularity of φ that is determined by Eq. (17). Such regions at a finite N should be as distant as possible from the points of singularity and we simply define them as vicinities of "middle" points $\bar{E}^{(i)}=\frac{1}{2}[E^{(i)}+E^{(i+1)}]$. The envelope curve may obviously be defined as the line passing through the points $[\bar{E}^{(i)},\ F(\bar{E}^{(i)})]$. The intersection with the straight line $F=E$ occurs at the only point stable with respect to evolution of this picture at $N\to\infty$. This intersection point is readily seen in Fig. 1.

Figure 1 shows the build up of singular points (vertical lines) with vanishing E and vice versa, a certain rarefaction of singularities in the vicinity of $E_{\rm SF}$. The former reflects growth of the density of states at the bottom of the exciton-pairs' band, whereas the latter is a usual effect of the "levels' repulsion." Note that the double-exciton shift for the CSFE level is positive as compared to the value $\mathcal{E}_{\rm SM}=0.627$. This is expected because the repulsion of the CSFE from the lower-lying crowded states of unbound excitons should be stronger than from the upper states having comparatively low density. One can also see in Fig. 1 some trend toward the concentration of singularity points $E^{(i)}$ at higher energies E. This is evidently a consequence of the density of states growth at the top of the exciton band.

IV. DOUBLE-MODE APPROXIMATION, FINITE THICKNESS, AND DISCUSSION

In general, the larger N is the more accurate the calculation of $\varphi(q)$ and E is, i.e., the envelope curve in Fig. 1 becomes discernible and may be drawn only at considerable N. At the same time, the analysis reveals that the intersection point with the $F=E$ line is rather stable and only weakly depends on N. This feature prompts us to consider the case $N=1$, where double-exciton states mixed with $\mathcal{Q}^\dagger_{0\bar{1}\mathbf{q}}|0\rangle$ are modeled by a single vector $|\mathrm{DX},1\rangle$. Actually, the $N=1$ approximation for the problem determined by Eqs. (13), (14), and (17) is equivalent to a variational procedure within the basis of two orthogonal trial states $|X_0\rangle^{(1)}=\mathcal{Q}^\dagger_{0\bar{1}0}|0\rangle+A_1|\mathrm{DX},1\rangle$ and $|X_0\rangle^{(2)}=A_1\mathcal{Q}^\dagger_{0\bar{1}0}|0\rangle-|\mathrm{DX},1\rangle$. If calculating

$$E_i(A_1)=\frac{{}^{(i)}\langle X_0|\hat{H}_{\rm int}|X_0\rangle^{(i)}}{{}^{(i)}\langle X_0|X_0\rangle^{(i)}},$$

where $i=1,2$, and minimizing either of these two values finding thereby the parameter $A_1=\mathcal{A}$ at which the minimum takes place, then the correlation part E of the CSFE energy is determined by the largest one of the two energies, $E_1(\mathcal{A})$ and $E_2(\mathcal{A})$. Namely $E=\max[E_1(\mathcal{A}),E_2(\mathcal{A})]-E^{\rm int}_{\rm vac}$ ($E^{\rm int}_{\rm vac}=\langle 0|\hat{H}_{\rm int}|0\rangle$ denotes the correlation part of the ground-state energy.)

After minor manipulations at $N=1$, we find that this simple double-mode approximation (DMA) reduces our problem to the secular equation,

$$\det|(E-\mathcal{E}_i)\delta_{ik}+(1-\delta_{ik})\mathcal{D}_{ik}|=0 \tag{19}$$

(indexes i and k are 1 or 2), where $\mathcal{E}_1=\int_0^\infty qdqV(q)\epsilon(q)$, $\mathcal{E}_2=\mathcal{E}_{\rm SM}$, and $\mathcal{D}_{12}\equiv\mathcal{D}_{21}=\int_0^\infty qdqV(q)d(q)$, with $\epsilon=2q^2(1-q^2)^2e^{-3q^2/2}+\frac{1}{2}(4-5q^2+q^4)e^{-q^2}+\frac{1}{16}(q^2-4)^3e^{-3q^2/4}+(2-q^2/2)e^{-q^2/2}$ and $d=q^2(q^2-1)e^{-q^2}$. Only the largest root of secular equation (19) has physical meaning. In the ideal 2D case, we easily obtain the DMA correlation energy of the CSFE $E_{\rm SF}=0.766$. Comparing this result with Fig. 1, we conclude that even the DMA works rather well.

Figure 2 shows the CSFE correlation energy calculated

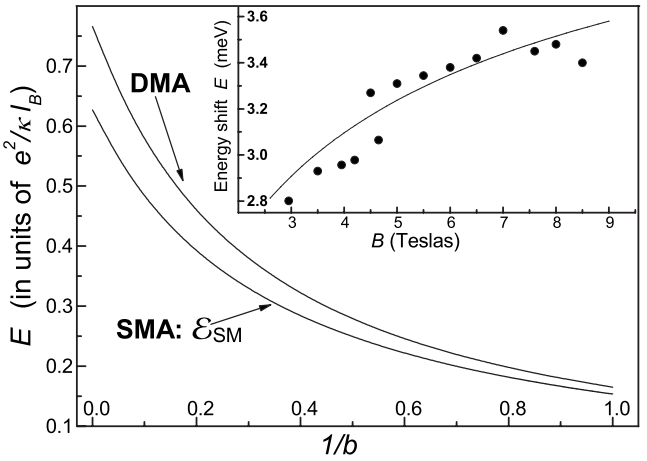

FIG. 2. Main picture: DMA and SMA correlation energies in dimensionless units against parameter b. Inset: DMA energy against the magnetic field when $b=5.45B^{-1/2}$ ($b=0.213l_B/\mathrm{nm}$; l_B in nm's, B in Teslas); symbols are experimental data for the 25 nm quantum wells (Ref. 11).

within the DMA and employing the SM approximation (SMA), if the vertex V for a real 2DEG is defined as $V=\mathcal{F}_b(q)/q$ with the formfactor,[1,8]

$$\mathcal{F}_b(q)=\frac{1}{8}\left(1+\frac{q}{b}\right)^{-3}\left[8+9\frac{q}{b}+3\left(\frac{q}{b}\right)^2\right].$$

Here $b=b_0 l_B$ is a dimensionless parameter corresponding to dimensionless q. b_0 is considered to be independent of the magnetic field. It is seen that additional shift of the CSFE energy determined by the DMA, being about 15% in the strict 2D limit (i.e., in the $b\to\infty$ case), becomes smaller ($\sim$5–6%) in real samples. This difference is not observable experimentally.[11] Meanwhile, the DMA results are in good agreement with experimental data, where the CSFE correlation energy is measured as a function of magnetic field, see inset in Fig. 2. The chosen value $b_0=0.213/\mathrm{nm}$ is quite consistent with the available wide quantum wells.[11]

In conclusion, we note that preliminary analysis indicates that the studied double-exciton correction should be more substantial in the case of a fractional filling, e.g., at $\nu=1/3$. Moreover, contrary to the SMA shifting of the energy to lower values as compared to a HF result,[8] the approach, taking into account the double-exciton component, should lead to a considerable positive shift in the CSFE correlation energy.

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APPENDIX A: EXCITONICALLY DIAGONALIZABLE PART OF THE HAMILTONIAN

If using ER, the relevant operators of the excitonically diagonalizable part in our case are

$$\hat{H}_{\mathrm{ED}}=\sum_{a=0,\bar{0},1,\bar{1}}\hat{H}_a+\sum_{ab=0\bar{0},01,\bar{0}1,0\bar{1}}\hat{H}_{ab},$$

where

$$\hat{H}_a=\frac{1}{2}\sum_{\mathbf{q}}V(q)h_{aa}^2(\mathbf{q})(\mathcal{Q}_{aa\mathbf{q}}^{\dagger}\mathcal{Q}_{aa\mathbf{q}}-N_{\phi}^{-1/2}\mathcal{Q}_{aa0}^{\dagger}),$$

$$\hat{H}_{ab}=\sum_{\mathbf{q}}V(q)[h_{aa}(\mathbf{q})h_{bb}(\mathbf{q})\mathcal{Q}_{aa\mathbf{q}}^{\dagger}\mathcal{Q}_{bb\mathbf{q}}+|h_{ab}(\mathbf{q})|^2\delta_{\sigma_a,\sigma_b}(\mathcal{Q}_{ab\mathbf{q}}^{\dagger}\mathcal{Q}_{ab\mathbf{q}}-N_{\phi}^{-1/2}\mathcal{Q}_{bb0}^{\dagger})],$$

and

$$h_{ab}=\left(\frac{n_a!}{n_b!}\right)^{1/2}\left(\frac{iq_x+q_y}{\sqrt{2}}\right)^{n_b-n_a}L_{n_a}^{n_b-n_a}(q^2/2)e^{-q^2/4}$$

(L_i^j is Laguerre polynomial).

APPENDIX B: THE $\mathbf{q}\neq\mathbf{0}$ CASE

For reference, we write out Eqs. (11) and (12) in the $\mathbf{q}\neq 0$ case,

$$[E_q-\mathcal{E}_{\mathrm{sw}}(|\mathbf{q}/2-\mathbf{p}|)-\mathcal{E}_{\mathrm{mp}}(|\mathbf{q}/2+\mathbf{p}|)]\varphi_q(\mathbf{p})-\frac{1}{\sqrt{2}}\{g_q(\mathbf{p})\}$$
$$=(2\pi)^{-1}\int d\mathbf{s}\varphi_q(\mathbf{s})\{[U_{00}(|\mathbf{p}-\mathbf{s}|)-\tilde{U}_{01}(|\mathbf{q}/2+\mathbf{s}|)]e^{i(\mathbf{p}\times\mathbf{s})_z}+U_{01}(|\mathbf{p}-\mathbf{s}|)e^{i(\mathbf{s}\times\mathbf{p})_z}-U_{00}(|\mathbf{p}-\mathbf{s}|)e^{i(\mathbf{q}\times(\mathbf{s}-\mathbf{p}))_z/2}-U_{01}(|\mathbf{p}-\mathbf{s}|)e^{i(\mathbf{q}\times(\mathbf{p}-\mathbf{s}))_z/2}\},$$

and $E_q=\mathcal{E}_{01}(q)+\frac{1}{\pi\sqrt{2}}\int d\mathbf{p}g_q^*(\mathbf{p})\varphi_q(\mathbf{p})$, where $g_q(\mathbf{p})=\tilde{U}_{01}(|\mathbf{p}+\mathbf{q}/2|)e^{i(\mathbf{p}\times\mathbf{q})_z/2}$, $\tilde{U}_{01}=V(q)|h_{01}(\mathbf{q})|^2$, and $U_{n_an_b}=V(q)h_{ab}^2(\mathbf{q})$ (see notations of Appendix A; notation $\{g_q\}$ in the left-hand side means the antisymmetry transform defined in Sec. II. If $\mathbf{q}$ is chosen parallel to $\hat{y}$, then $\varphi_q(\mathbf{p})$ is an even function with respect to the replacement $p_x\to -p_x$. The SM result $\mathcal{E}_{01}(q)$ was calculated in Ref. 8.

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